THE 3-SPACE METHOD FOR SOLVING PARABOLIC ROOTS

FREDERICK HERRMANN
MAKUA LANI CHRISTIAN ACADEMY
KAILUA-KONA, HAWAII

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In my paper *Mathematica Exploratio I* (June 2021), I demonstrated the 3-Space Method for solving the roots of a parabola using hyperbolic paraboloids. This paper demonstrates solving parabolic roots with circular paraboloids.

The 3-Space Method Using Circular Paraboloids

$$-az^2 = ax^2 + bx + c$$

DEFINITIONS

$y = ax^2 + bx + c$	Standard form of a vertical parabola.
x_m	The x-value of a parabola's line of symmetry; the x-value where the parabola has either its maximum or minimum y-value.
y_m	The maximum or minimum y-value of the parabola.
$V(x_m, y_m)$	The vertex of the parabola.
(x, y, z)	The 3-space dimension. For this work, the y-direction is always considered vertical.
x_0	The "roots" of an equation; the x-values of an equation where $y = 0$; the solutions to the quadratic formula.

3-Space Method

Consider the parabola

$$y = ax^2 + bx + c$$

Complete the following steps:

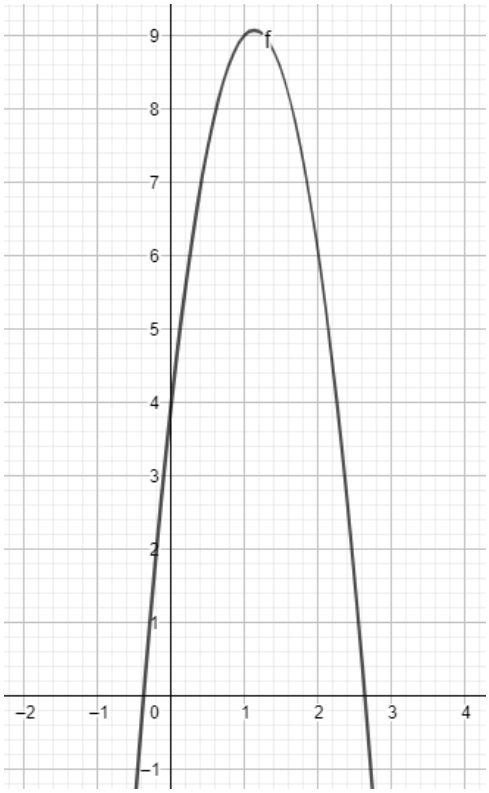
	$y = ax^2 + bx + c$
Add the z-term to create the paraboloid.	$y = ax^2 + bx + c + az^2$
Set $y = 0$.	$0 = ax^2 + bx + c + az^2$
Move the z-term to the left side of the equation.	$-az^2 = ax^2 + bx + c$
Find x_m.	$x_m = -\frac{b}{2a}$
Place the x_m into the equation.	$az^2 = a{x_m}^2 + bx_m + c$
Sum the right side.	$az^2 = y_m$
You now have the vertex.	(x_m, y_m)
Solve for z.	$z = \sqrt{\frac{y_m}{a}}$
The solutions are:	$x_0 = x_m \pm z$

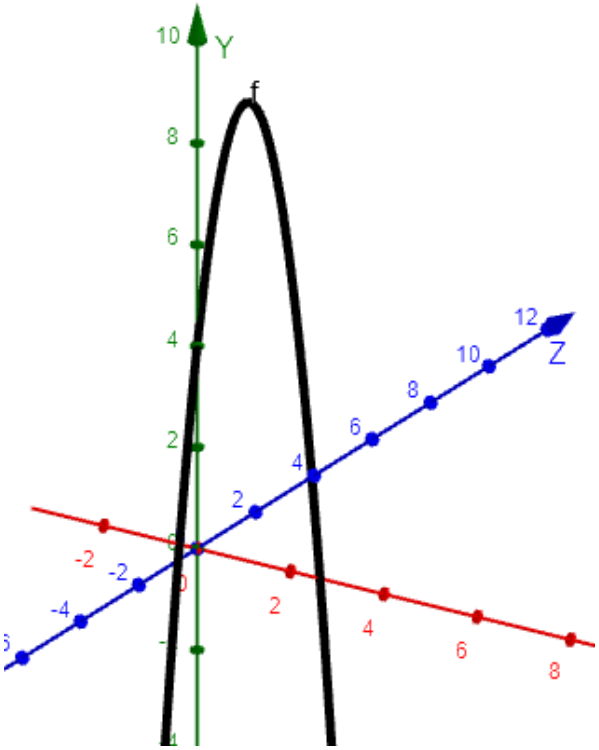
Example with Real Solutions

The parabola.	$y = -4x^2 + 9x + 4$
Add $-4z^2$.	$y = -4x^2 + 9x + 4 - 4z^2$
Let $y = 0$.	$0 = -4x^2 + 9x + 4 - 4z^2$
Move $4z^2$ to the left side of the equation.	$4z^2 = -4x^2 + 9x + 4$
Find x_m.	$x_m = -\frac{b}{2a} = \frac{9}{8} = 1.125$
Place x_m into the equation and solve.	$4z^2 = -4(1.125)^2 + 9(1.125) + 4 = 9.0625$
You now have the vertex.	$V(1.125, 9.0625)$
Solve for z.	$z = \sqrt{\frac{9.0625}{4}} = \pm 1.505$
The solutions are:	$x_0 = 1.125 \pm 1.505 = 2.63\ and - 0.38$

Example with Real Solutions: Images

The parabola.	$y = -4x^2 + 9x + 4$





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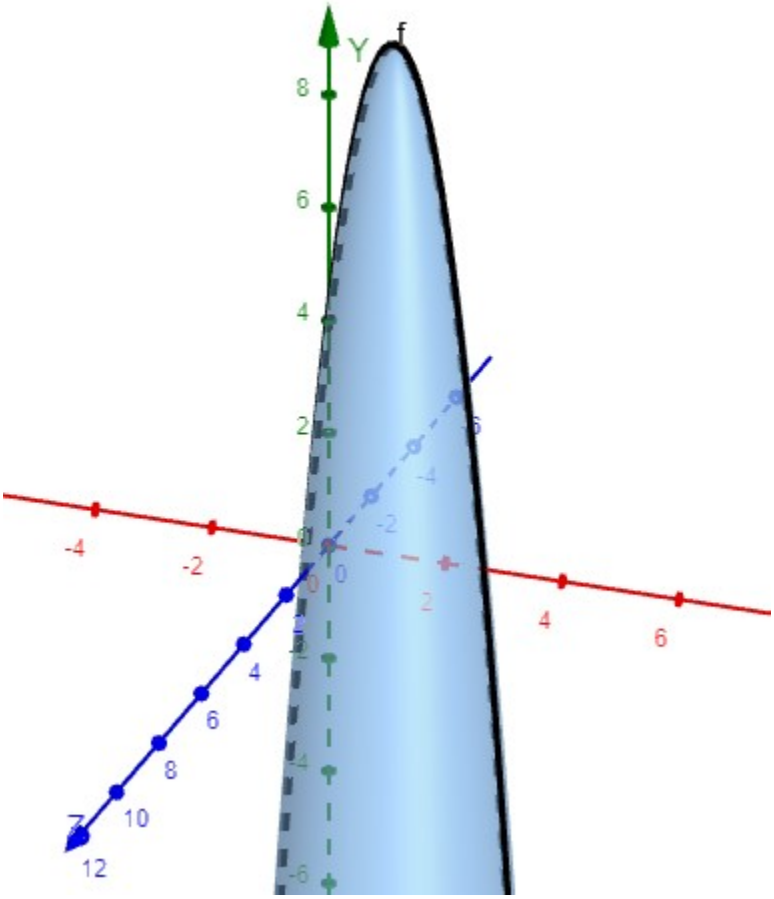
Add $-4z^2$.	$y = -4x^2 + 9x + 4 - 4z^2$



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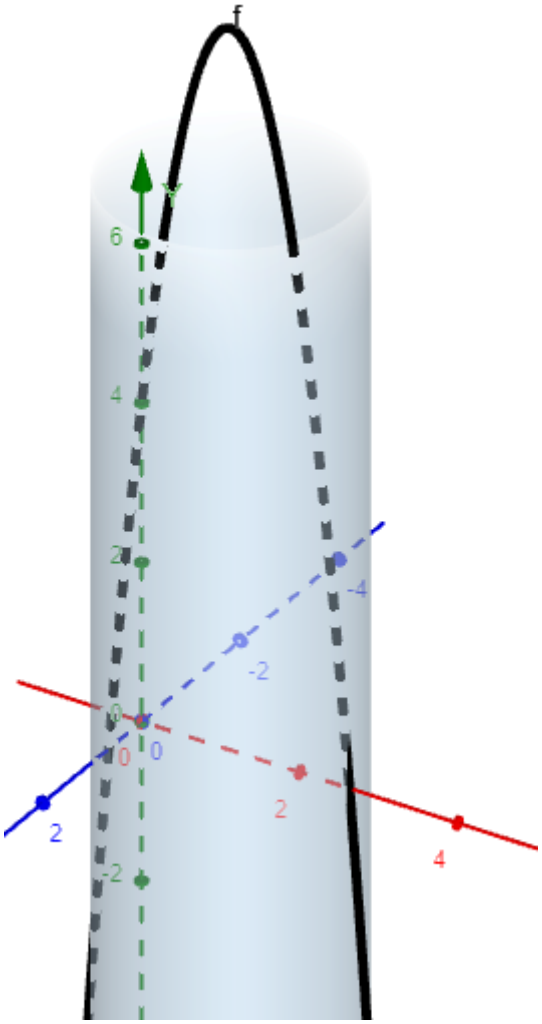
Let $y = 0$.	$0 = -4x^2 + 9x + 4 - 4z^2$
Move $4z^2$ to the left side of the equation.	$4z^2 = -4x^2 + 9x + 4$



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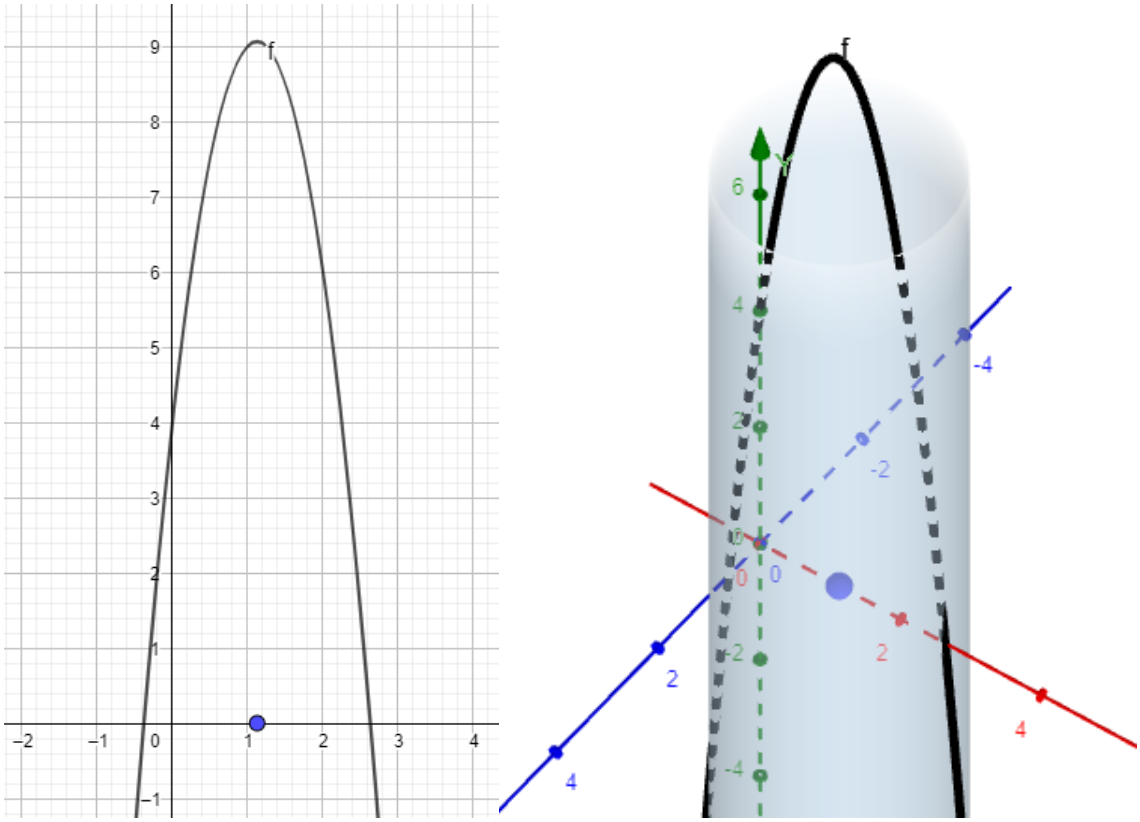
Find x_m.	$x_m = -\frac{b}{2a} = \frac{9}{8} = 1.125$



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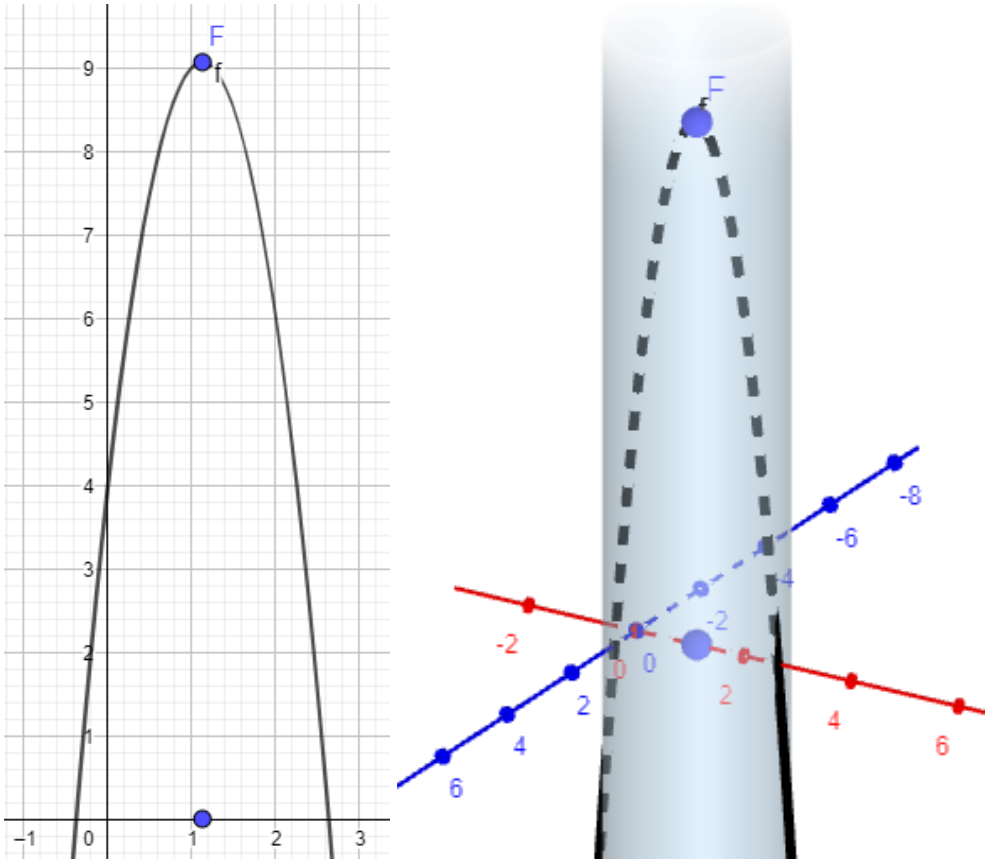
Place x_m into the equation and solve.	$4z^2 = -4(1.125)^2 + 9(1.125) + 4 = 9.0625$
You now have the vertex.	$V(1.125, 9.0625)$



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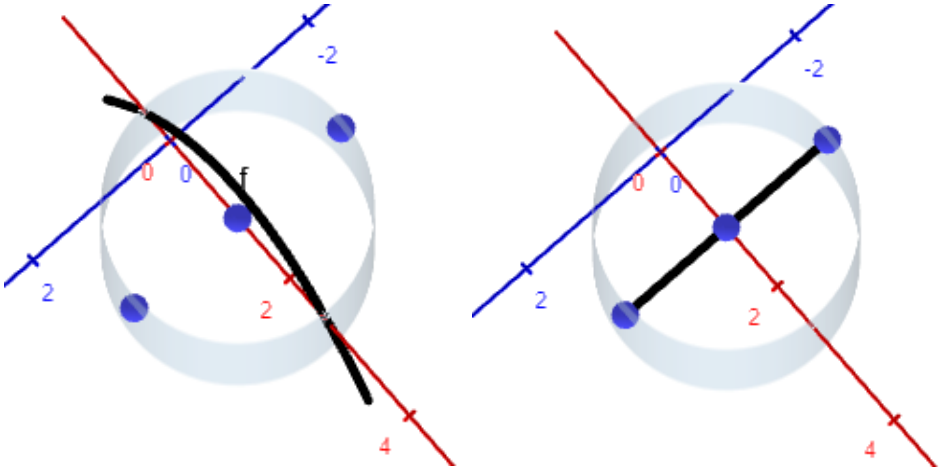
Solve for z.	$z = \sqrt{\frac{9.0625}{4}} = \pm 1.505$



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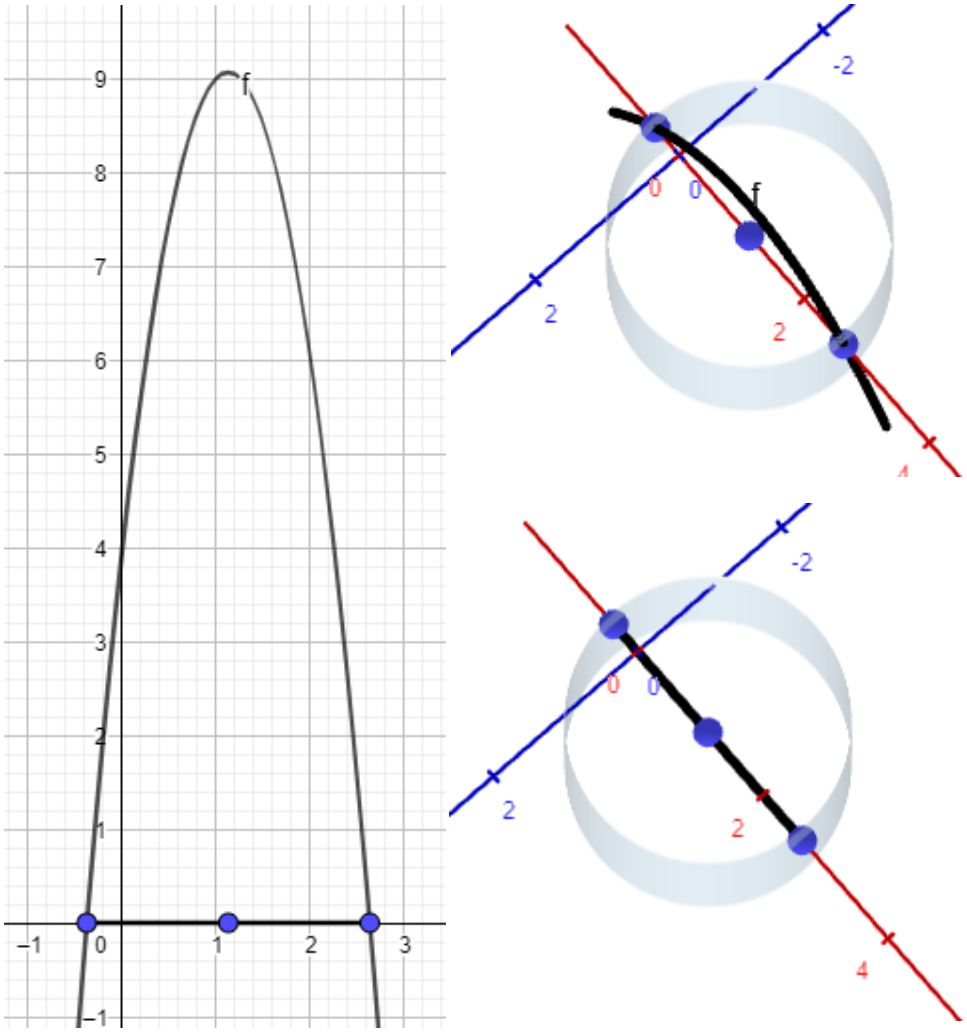

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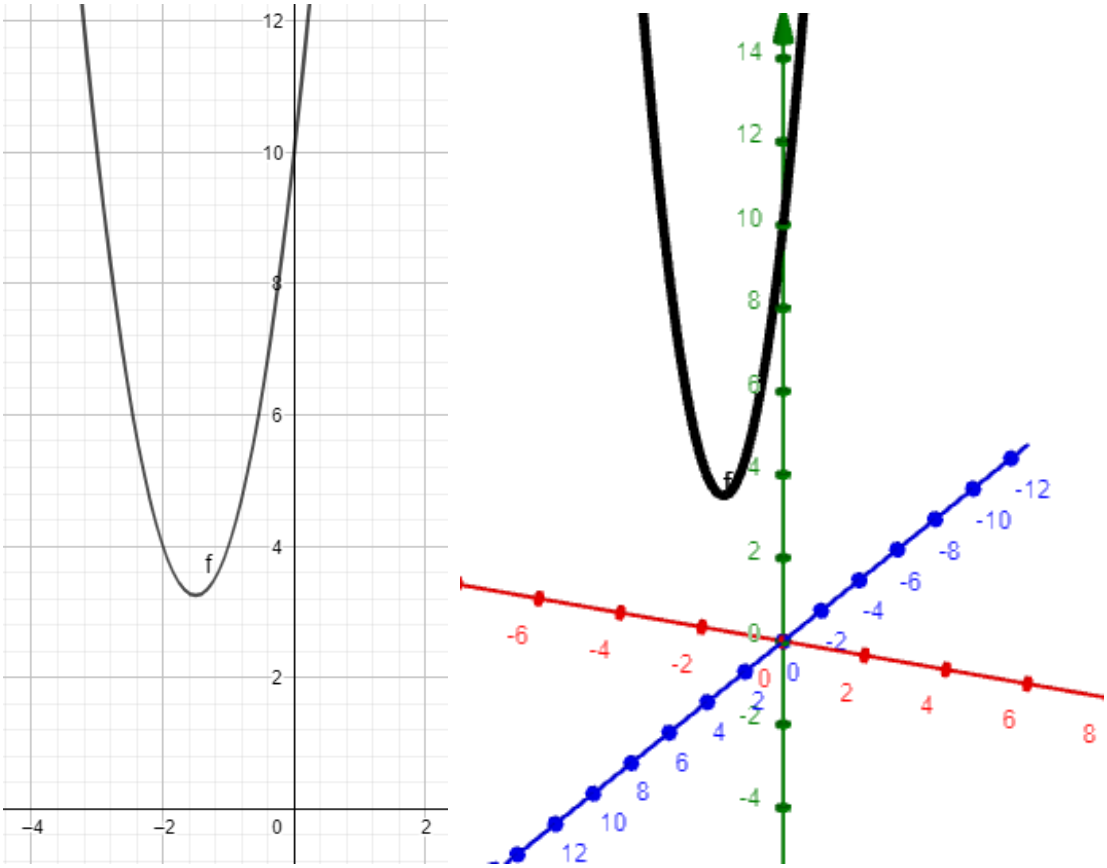
Example with Imaginary Solutions

The parabola.	$y = 3x^2 + 9x + 10$
Add $3z^2$.	$y = 3x^2 + 9x + 10 + 3z^2$
Let $y = 0$.	$0 = 3x^2 + 9x + 10 + 3z^2$
Move $3z^2$ to the left side of the equation.	$-3z^2 = 3x^2 + 9x + 10$
Find x_m.	$x_m = -\frac{b}{2a} = -\frac{9}{6} = -1.5$
Place x_m into the equation and solve.	$-3z^2 = 3(-1.5)^2 + 9(-1.5) + 10 = 3.25$
You now have the vertex.	$V(-1.5, 3.25)$
Solve for z.	$z = \sqrt{\frac{3.25}{-3}} = \pm 1.041i$
The solutions are:	$x_0 = -1.5 \pm 1.041i$

Example with Imaginary Solutions: Images

The parabola.	$y = 3x^2 + 9x + 10$



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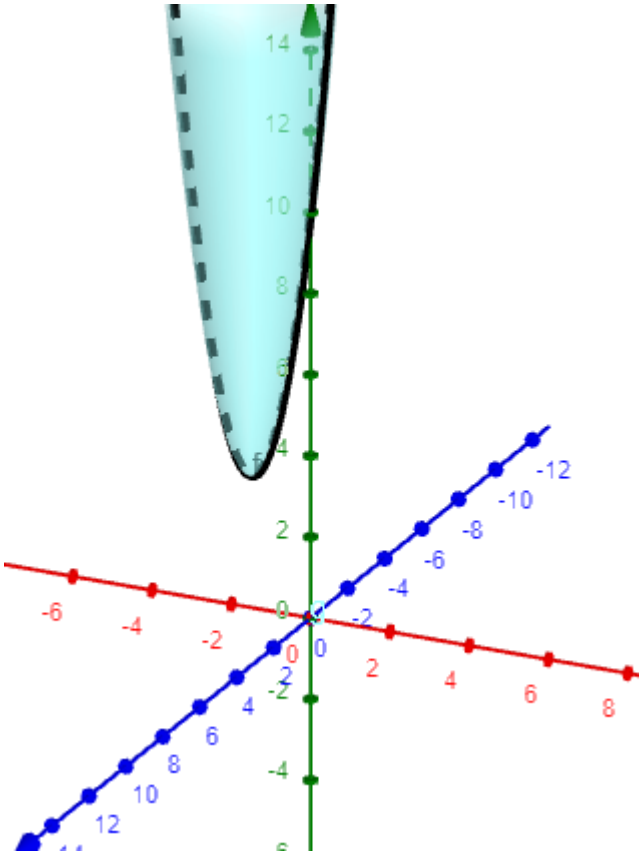
Add $3z^2$.	$y = 3x^2 + 9x + 10 + 3z^2$



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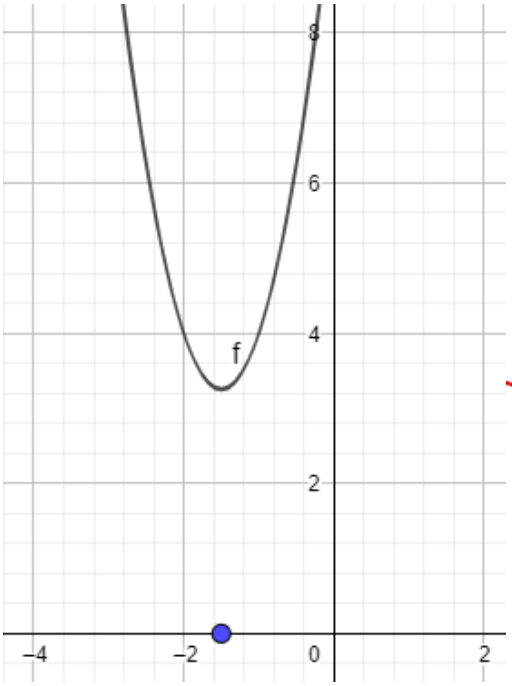
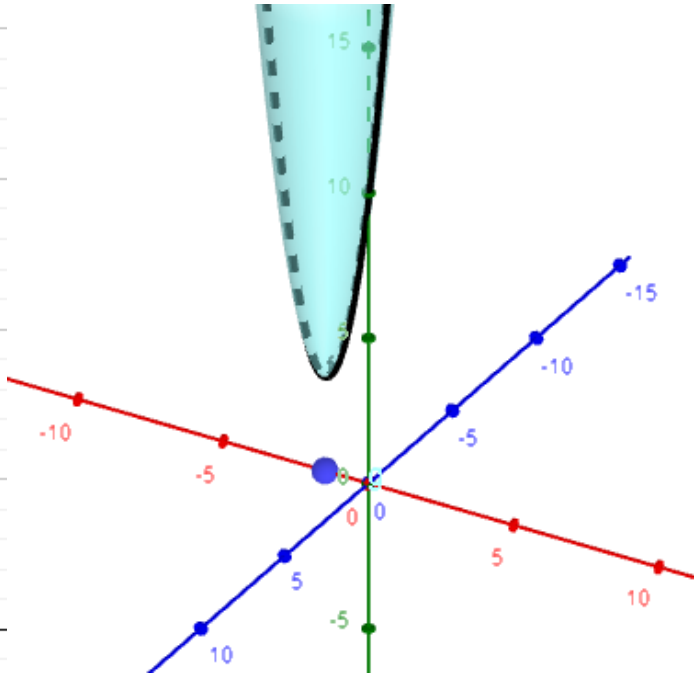
Let $y = 0$.	$0 = 3x^2 + 9x + 10 + 3z^2$
Move $3z^2$ to the left side of the equation.	$-3z^2 = 3x^2 + 9x + 10$
Find x_m.	$x_m = -\frac{b}{2a} = -\frac{9}{6} = -1.5$





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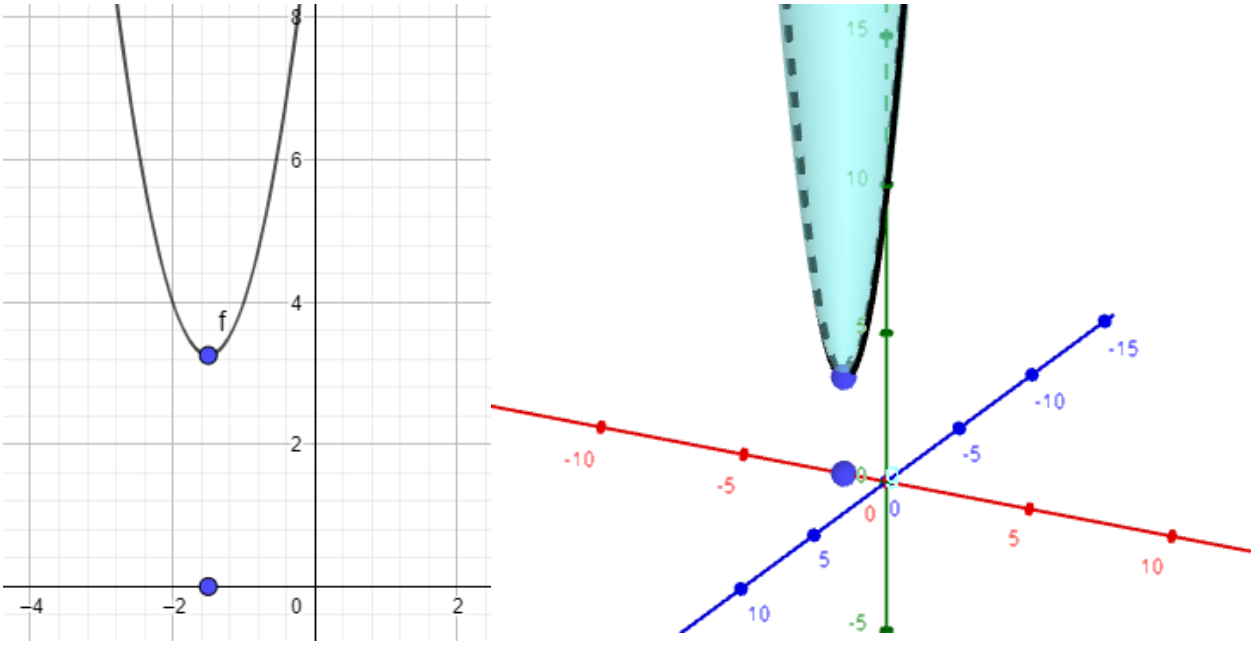
Place x_m into the equation and solve.	$-3z^2 = 3(-1.5)^2 + 9(-1.5) + 10 = 3.25$
You now have the vertex.	$V(-1.5, 3.25)$



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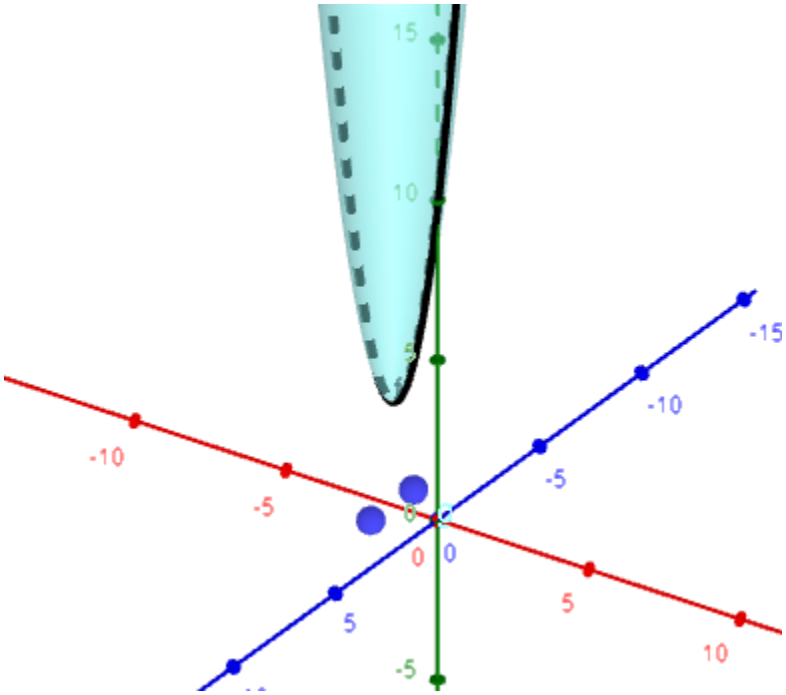
Solve for z.	$z = \sqrt{\frac{3.25}{-3}} = \pm 1.041i$
The solutions are:	$x_0 = -1.5 \pm 1.041i$



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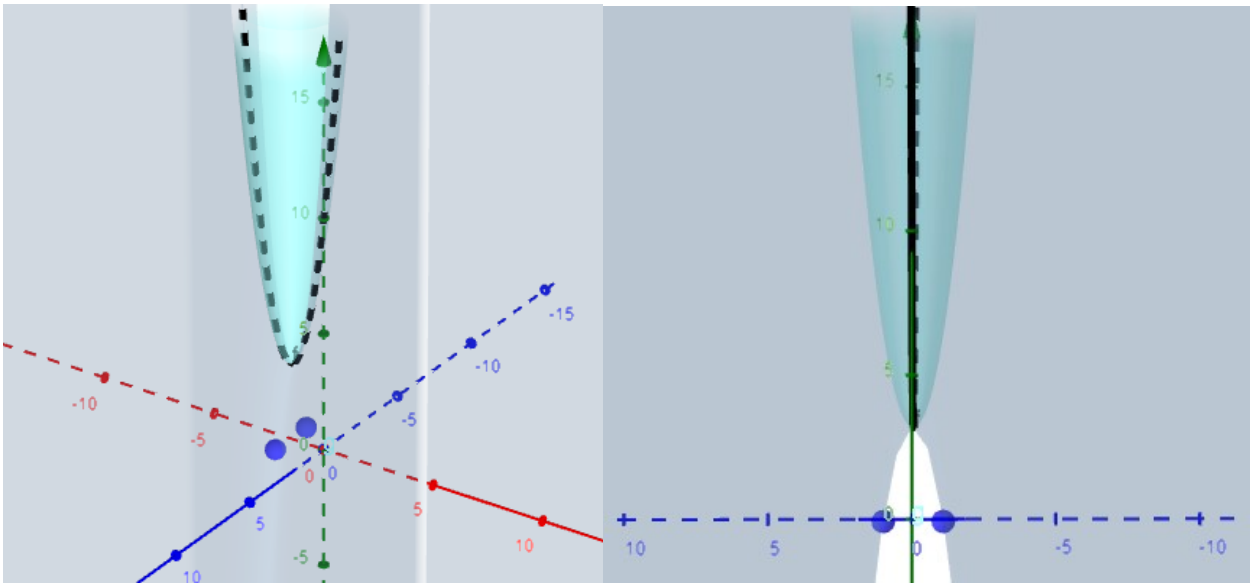
Solutions Images with Hyperbolic Paraboloid in Gray



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